Worksheet 10 - Electromagnetic Radiation and the Bohr Atom

Light is known to have the wave-like properties of frequency (v) and wavelength (λ). These are illustrated below. The x-axis is a measure of time. The distance between the peaks is called the wavelength and the number of waves per unit time (1 second in this example) is called the number of cycles. The first wave pattern has 2 cycles per second, the middle example has 4 cycles per second and the example on the right has 8 cycles per second. This is the frequency of the wave, and has the units of hertz, Hz (cycles/s).

\[ \text{1 second} \]

As the frequency increases, the wavelength decreases.

In Electromagnetic radiation (light) these are related by the equation:

\[ c = \lambda v \]

where \( c \) = the speed of light, 2.998 x 10^8 m/s, \( \lambda \) = wavelength (m) and \( v \) = frequency (s\(^{-1}\) or Hz). The electromagnetic spectrum (EMS) is shown below.

Which color of visible light has the shortest wavelength? Which radiation has wavelengths longer than visible light? \( \text{Violet} \text{, IR,} \)

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### Electromagnetic Spectrum

<table>
<thead>
<tr>
<th>0.0001 nm</th>
<th>0.01 nm</th>
<th>10 nm</th>
<th>1000 nm</th>
<th>0.01 cm</th>
<th>1 cm</th>
<th>1 m</th>
<th>100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma rays</td>
<td>X-rays</td>
<td>UV-violet</td>
<td>Infrared</td>
<td>Radio waves</td>
<td>Radar TV FM AM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Visible light
1. The wavelength of green light is about 522 nm. What is the frequency of this radiation?

\[ C = \lambda \nu \]
\[ 2.998 \times 10^8 \text{ m/s} = \nu \times 522 \times 10^{-9} \text{ m} \]
\[ \nu = 5.74 \times 10^{14} \text{ s}^{-1} \]

Planck recognized that energy is quantized and related the energy of radiation (emitted or absorbed) to its frequency.

\[ \Delta E = n \ h \ \nu = n \ h \ c \]

where \( n \) is integer and \( h = \) Planck's constant \( = 6.626 \times 10^{-34} \text{ J s} \)

2. What is the wavelength of a photon that has a frequency of \( 2.10 \times 10^{14} \text{ Hz} \) ? Answer in nm and determine what type of radiation this is.

\[ \nu = 2.10 \times 10^{14} \text{ Hz (s}^{-1}) \]
\[ C = \lambda \nu \]
\[ 2.998 \times 10^8 \text{ m/s} = 2.10 \times 10^{14} \text{ s}^{-1} \times \lambda \]
\[ \lambda = 1.43 \times 10^{-6} \text{ m} \times \frac{1 \times 10^9 \text{ nm}}{1 \text{ m}} = 1427 \text{ nm} \]

IR.

3. Which of the following are directly related?

a) energy and wavelength - reversely related
b) wavelength and frequency - directly related
c) frequency and energy - directly related

4. A classical radio station broadcasts at 93.5 MHz \( (M = 10^6) \). Find the wavelength of this radiation, in meters, and the energy of one of these photons, in J. What type of radiation is this?

\[ 93.5 \times 10^6 \text{ Hz} = \nu \]
\[ C = \lambda \nu \]
\[ 2.998 \times 10^8 \text{ m/s} = 93.5 \times 10^6 \text{ Hz} \times \lambda \]
\[ \lambda = \frac{2.998 \times 10^8 \text{ m/s}}{93.5 \times 10^6 \text{ Hz}} = 3.21 \text{ m} \]

Radio wave \( (\text{FM}) \)

5. What is the energy of a photon with:

a) a wavelength of 827 nm? What type of radiation is it?

\[ \Delta E = h \nu = h \frac{c}{\lambda} = \frac{6.626 \times 10^{-34} \text{ Js}}{827 \times 10^{-9} \text{ m}} = \frac{2.40 \times 10^{-19} \text{ J}}{827 \text{ nm} \lambda \text{ m} \text{ km}} \]

\( 827 \text{ nm} \approx \lambda \text{ m} \text{ km} \)

b) a wavelength of 1 nm? What type of radiation is it?

\[ \Delta E = h \frac{c}{\lambda} = \frac{6.626 \times 10^{-34} \text{ Js}}{2.998 \times 10^8 \text{ m/s} \times 1 \times 10^{-9} \text{ m} \text{ km}} = 1.98 \times 10^{-19} \text{ J} \]

\( X \)-ray
Bohr applied this concept to the **line spectra** of elements. When elements are excited they emit radiation at fixed wavelengths. He proposed that only certain **energy levels** are allowed within the structure of an atom. Electrons are allowed to move between these energy levels. The light emitted by the elements is a measure of the energy gap between the two electronic states.

For the hydrogen atom,

\[
\Delta E = -R_H \frac{Z^2}{n_2^2} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)
\]

\[R_H = \text{Rydberg constant} = 2.178 \times 10^{-18} \text{J}\]

\[Z = \text{nuclear charge} = 1 \text{ for H, 2 for He}\]

6. Calculate the \(\Delta E\) for the \(n = 4\) to the \(n = 2\) transition in hydrogen. Where on the EMS would this appear? What does the sign mean?

\[
\Delta E = -2.178 \times 10^{-18} \text{J} \cdot \frac{1}{2^2} \left( \frac{1}{4^2} - \frac{1}{2^2} \right)
\]

\[
\Delta E = -4.08 \times 10^{-19} \text{J}
\]

\[
\Delta E = \frac{\hbar c}{\lambda} = \frac{6.626 \times 10^{-34} \text{Js} \cdot 2.998 \times 10^8 \text{m/s}}{\lambda}
\]

\[\lambda = 4.86 \times 10^{-3} \text{m} = 486 \text{ nm} = \text{blue green}\]

7. A hydrogen atom in its ground state absorbs light with a wavelength of 102.6 nm. Calculate the energy level of the resulting excited state \((n = ?)\).

\[
\Delta E = \frac{\hbar c}{\lambda} = \frac{6.626 \times 10^{-34} \text{Js} \cdot 2.998 \times 10^8 \text{m/s}}{102.6 \times 10^{-9} \text{m}}
\]

\[
\Delta E = 1.94 \times 10^{-18} = \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)^2 \frac{1}{2^2} \left( \frac{1}{4^2} - \frac{1}{2^2} \right)
\]

\[-1.88895 = \left( \frac{1}{n_f^2} - 1 \right)\]

\[\text{from } n_i = 1 \text{ to } n_f = 3\]

\[\frac{1}{n_f^2} = 0.333\]

\[n_f^2 = 3\]
Ionization energy is the energy required to completely remove an electron from an atom. This can be thought of as the transition between \( n = 1 \) and \( n = \infty \).

8. Calculate the energy needed to remove the electron from hydrogen in its ground state.

\[
\Delta E = -2.178 \times 10^{-18} \text{J} \cdot \frac{1}{2} \left[ \frac{1}{\infty^2} - \frac{1}{1^2} \right] \quad \frac{1}{\infty} = 0
\]

\[
\Delta E = \left( -2.178 \times 10^{-18} \text{J} \right) (-1)
\]

\[
\Delta E = 2.178 \times 10^{-18} \text{J}
\]

Since energy must be put in.

This is the energy to remove an electron from the ground state of hydrogen. What wavelength of light would work? Where is this on the EMS?

\[
\Delta E = \frac{hc}{\lambda} = \frac{2.178 \times 10^{-18} \text{J} \cdot 2.9979 \times 10^8 \text{m/s}}{\lambda}
\]

\[
\lambda = 9.12 \times 10^{-8} \text{m} = 91.2 \text{ nm}
\]

UV

9. What is the energy needed to remove the remaining electron from He\(^+\) in its ground state? Is it easier or harder to remove the electron from He\(^+\) than from H?

\[
\Delta E = 2.178 \times 10^{-18} \text{J} \cdot \frac{1}{2} \left[ \frac{1}{\infty^2} - \frac{1}{1^2} \right]
\]

\[
= 8.712 \times 10^{-19} \text{J}
\]

It is harder to remove from He\(^+\)

\[
8.712 \times 10^{-19} \text{J} = 6.626 \times 10^{-34} \text{J} \cdot s \cdot 2.998 \times 10^8 \text{m/s}
\]

\[
\lambda = \frac{2.12 \times 10^{-10} \text{m}}{1.2 \times 10^{-15} \text{m/s}} = 1.7 \text{ mm}
\]